Test 1 – Calc Emphasizing Proofs

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P1. Prove the following:

(*i*)

$$||x| - |y|| \leq |x - y| \leq |x| + |y|.$$

(*ii*)

$$\frac{1}{|x| + |y|} \leq \left| \frac{1}{x - y} \right| \leq \frac{1}{||x| - |y||} \qquad \text{if} \quad |x| \neq |y|.$$

(*iii*)

$$|x_1 + x_2 + \cdots + x_n| \leq |x_1| + |x_2| + \cdots + |x_n|.$$

P2. Find the following limits:

(*i*)

$$\lim_{x \to 1} \frac{1 - x}{1 - \sqrt{x}}.$$

(*ii*)

$$\lim_{x \to 0} \frac{\sqrt{2x + 1} - 1}{4x}.$$

(*iii*) Using the definition to prove that

$$\lim_{x \to \infty} \frac{\sin x}{x} = 0.$$

P3. Answer by yes or no for the following questions. If your answer is yes, prove it. If your answer is no, give a counter-example.

(*i*) If $\lim_{x \to a} f(x) = l$, then there exist $\delta > 0$ and $k > 0$ such that for all x satisfying $|x - a| < \delta$, we have that $|f(x)| \leq k$.

(*ii*) If $\lim_{x \to a} |f(x)| = |l|$, then $\lim_{x \to a} f(x) = l$ or $\lim_{x \to a} f(x) = -l$.