

Analysis I Fall 2013 – NYUSH

In-Class Midterm (60%)

Problem 1. (10 points) Find the domain of the function

$$f(x) = \frac{\sqrt{x+1}}{x-2}.$$

Problem 2. (16 points) Find the following limits and briefly explain why.

(a)

$$\lim_{n \rightarrow \infty} \frac{x_n}{n^a}.$$

Here $a > 0$ and $\{x_n\}$ is a bounded sequence of real numbers.

(b)

$$\lim_{x \rightarrow 0} \frac{x^2 + 3x - 2}{\sqrt{1-x^2}}.$$

Problem 3. (10 points) Show that on the interval $[0, \frac{\pi}{2}]$ there is a root of the equation

$$x = \cos x.$$

Problem 4. (14 points) Let $a_1 = 0, a_{n+1} = \sqrt{5 + 2a_n}$ for $n = 1, 2, \dots$. Show (by induction) that $\{a_n\}$ is monotone increasing. Find the value of $\lim_{n \rightarrow \infty} a_n$.

Problem 5. (10 points) Use the ε - δ terminology to show that the function

$$f(x) = \frac{1}{2+5x}$$

is continuous at $x = 1$.