Homework 9 – Calc Emphasizing Proofs

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Page 239 problem 1. (i)$f^{-1}(x) = (x-1)^{1/3}$.

(ii) $f^{-1}(x) = x^{1/3} + 1$. (If $y = f^{-1}(x)$, then $x = f(y) = (y-1)^3$, so $y = 1 + x^{1/3}$.)

(iii)

$$f^{-1}(x) = \begin{cases} x, & x \text{ rational}, \\ -x, & x \text{ irrational}. \end{cases}$$

(iv)

$$f^{-1}(x) = \begin{cases} (-x)^{1/2}, & x \leq 0, \\ (1-x)^{1/3}, & x > 1. \end{cases}$$

(If $y = f^{-1}(x)$, then

$$x = f(y) = \begin{cases} -y^2, & y \geq 0, \\ 1-y^3, & y < 0. \end{cases}$$

Since $-y^2 \leq 0$ if $y \geq 0$ and $1 - y^3 > 1$ if $y < 0$, we have $y = (-x)^{1/2}$ for $x \leq 0$ and $y = (1-x)^{1/3}$ for $x > 1$.)

(v)

$$f^{-1}(x) = \begin{cases} a_n, & x = a_1, \\ a_i, & x = a_{i+1}, \ (i = 1, 2, \cdots, n-1), \\ x, & x \neq a_1, a_2, \cdots, a_n. \end{cases}$$

(vi) $f^{-1}(x) = x - [x/2]$ for $[x]$ even. (If $y = f^{-1}(x)$, then

$$x = f(y) = y + [y] = y + n \qquad \text{for } \ n \leq y < n+1.$$

Thus

$$2n \leq x < 2n+1,$$

and

$$y = x - n = x - [x/2].)$$

(vii) $f^{-1}(x) = f(x)$.

(viii)

$$f^{-1}(x) = \begin{cases} \frac{-1+\sqrt{1+4x^2}}{2x}, & x \neq 0, \\ 0, & x = 0. \end{cases}$$

(If $y = f^{-1}(x)$, then $x = f(y) = y/(1-y^2)$. So $xy^2 + y - x = 0$. If $x = 0$, then $y = 0$. If $x \neq 0$, then

$$y = \frac{-1+\sqrt{1+4x^2}}{2x} \quad \text{or} \quad y = -1 - \sqrt{\frac{1+4x^2}{2x}}.$$

The first possibility is the correct one, since x and y must have the same sign.)

Page 240 problem 6. Suppose that $f(x_1) = f(x_2)$, i.e. $\frac{ax_1+b}{cx_1+d} = \frac{ax_2+b}{cx_2+d}$ and then

$$(ad - bc)(x_1 - x_2) = 0.$$

So f is one-one if and only if $ad - bc \neq 0$. Moreover,

$$f^{-1}(x) = \frac{dx - b}{a - cx} \qquad (x \neq \frac{a}{c}).$$

Page 240 problem 7.

(i) Those intervals $[a, b]$ which are contained in the interval $(-\infty, 0]$ or in $(0, 2)$ or in $[2, \infty)$, since these are the intervals on which f is increasing or decreasing.

(ii) Any interval $[a, b]$, since f is increasing.

(iii) Those intervals $[a, b]$ which are contained in the interval $(-\infty, 0]$ or in $[0, \infty)$, since these are the intervals on which f is increasing or decreasing.

(iv) Those intervals $[a, b]$ which are contained in the interval $(-\infty, -1 - \sqrt{2}]$ or in $(-1 - \sqrt{2}, -1 + \sqrt{2})$ or in $[-1 + \sqrt{2}, \infty)$, since these are the intervals on which f is increasing or decreasing.